PROPAGATION OF A FAN-SHAPED JET IN A BOUNDED VOLUME

M. D. Tarnopol'skii

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Analytical formulas are derived for calculating the velocities of a flat fan-shaped jet propagating in a bounded volume, starting from the premise that the radial velocity components in a constrained flat jet are less than in a free flat jet by the magnitude of the "velocity shift."

An examination is made of the circulation of a liquid (or gas) in a bounded volume of cylindrical shape, due to an immersed fan-shaped jet. The fan-shaped jet is formed by feeding a liquid (or gas) through an aperture in the center of the upper end of a cylinder onto a disk mounted transverse to the stream. The jet lies flat on the upper plate; starting at some section, the jet unrolls and forms an intersecting stream located between the active part of the jet and the lower plate of the cylinder. The liquid (or gas) is removed from the volume through an aperture in the center of the lower plate. The radius of the cylinder ends is appreciably greater than its height.

An examination is made of the flow due to a point source with zero flow rate of liquid (or gas), but with a finite initial impulse.

We shall make use of the premises put forward in papers dealing with axisymmetric and plane flows of a jet in a finite volume [1–3].

We shall assume that the distribution of radial velocity components in a flat constrained fan-shaped jet obeys the same law as in a flat free jet but that the velocities differ by the magnitude of the "velocity shift." The velocity shift is the same in each cylindrical section but varies from section to section (Fig. 1).

We shall neglect friction arising from motion of the liquid (gas) at the bounding surfaces.

The analytical expression of these assumptions is the equation for the radial velocity component of the liquid or gas at an arbitrary point of a constrained jet

$$U' = U - U_*. \tag{1}$$

We shall find the value of each of the terms on the right of (1). The radial velocity component at an arbitrary point of a fan-shaped free jet is given by

$$U = U_x \exp\left[-\frac{1}{2}\left(\frac{y}{cx}\right)^2\right], \tag{2}$$

where U_x is the velocity on the axis of the free jet, as given by the formula in [4]

$$U_x = \frac{0.423}{\sqrt{c}}\sqrt{\frac{I_0}{\rho}}\,\frac{1}{x}. \tag{3}$$

The value of the velocity shift U_* may be determined from the continuity equation, according to which the amount of liquid (or gas) displaced into the active part of the jet in the direction of discharge is equal to the amount of liquid (or gas) in the intersecting stream

$$\int_0^{y_b} U'dF = -\int_{y_b}^{H} U'dF, \tag{4}$$

where dF is an element of area of the cylindrical section of the jet within which the velocity U' is the same:

$$dF = 2\pi x dy. \tag{5}$$

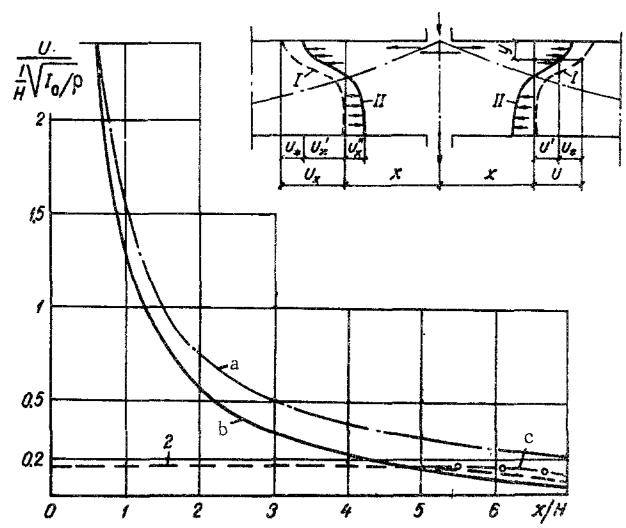

Fig. 1. Variation of velocity in a finite volume. On the graph: a is the velocity U_x on the axis of a flat free jet according to (3); b is the velocity U'_x in the plane in which the active part of the constrained jet lies, according to (8); c is the velocity shift U_* according to (6) in a constrained flat jet; d is the velocity U''_x according to (9) in the plane along which the intersecting stream propagates. The diagrams show the velocity profile: I—in a free jet; II—in a constrained jet.

Simultaneous solution of (1)–(5) gives the value of the velocity shift:

$$U_* = \frac{0.532\sqrt{c}}{H}\sqrt{\frac{I_0}{\rho}}\,\mathrm{erf}\left(\frac{H}{\sqrt{2}cx}\right). \tag{6}$$

Now Eq. (1) may be represented in the following form:

$$U' = U_x \exp\left[-\frac{1}{2}\left(\frac{y}{cx}\right)^2\right] - U_*, \tag{7}$$

where U_x and U^* are determined from (3) and (6).

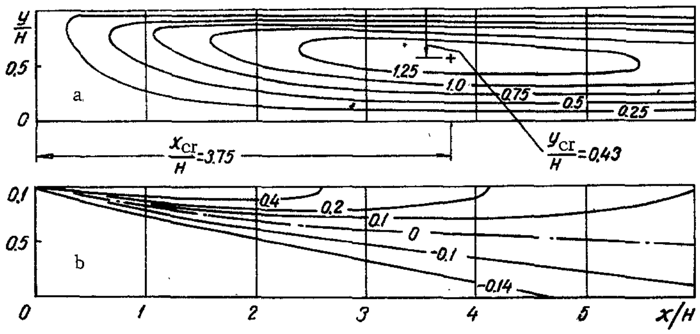

Fig. 2. Streamlines and flow isovels in a bounded volume: a—streamlines, the value $\varphi'/H(I_c/\rho)^{1/2}$ = const; b—isovels, the value $U'/(1/H)(I_0/\rho)^{1/2}$ = const.

The velocity of the liquid (or gas) in the immediate vicinity of the plane containing the active part of the jet is determined from (7) with y = 0:

$$U_x' = U_x - U_*. \tag{8}$$

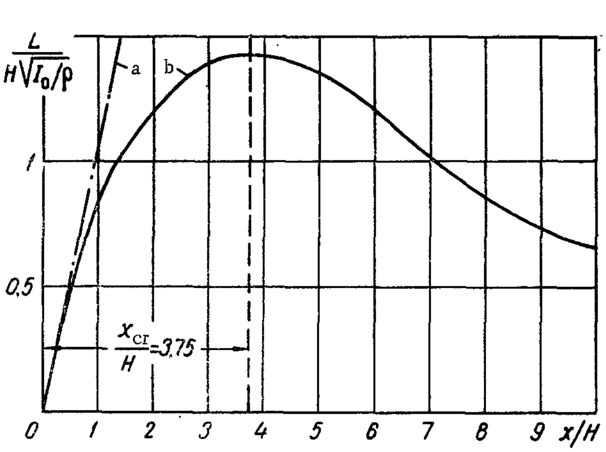

Fig. 3. Variation of flow rate: a—in a free jet L_x from (15); b—in a constrained jet L_x' from (16).

The velocity of the liquid (or gas) in the immediate vicinity of the plane along which the intersecting jet propagates is determined from (7) with y = H:

$$U_x'' = U_x \exp\left[-\frac{1}{2}\left(\frac{H}{cx}\right)^2\right] - U_*. \tag{9}$$

This velocity has a maximum on the axis of symmetry

$$(U_x'')_{\max} = \frac{0.152}{H}\sqrt{\frac{I_0}{\rho}}. \tag{10}$$

Graphs of relations (3), (6), (8), and (9), drawn for the value C = 0.082 of the experimental constant, are shown in Fig. 1.

We obtain the equation for surfaces of constant velocity from (7) by solving it with respect to y:

$$y = \sqrt{2}\,cx\sqrt{\ln\frac{U_x}{U' + U_*}}. \tag{11}$$

The equation of the surface separating the active part of the jet from the intersecting stream is obtained from (11) with U' = 0:

$$y_b = \sqrt{2}\,cx\sqrt{\ln\frac{U_x}{U_*}}. \tag{12}$$

The value of the stream function is determined from the formula

$$\psi' = \int_0^y U'dF. \tag{13}$$

After substitution of the values of U' and dF from (5) and (7) and subsequent integration, we obtain

$$\psi' = L_x \operatorname{erf}\left(\frac{y}{\sqrt{2}\,cx}\right) - 2\pi xyU_*, \tag{14}$$

where L_x is the mass flow rate of liquid (or gas) in the cylindrical section of the free jet, as determined from the formula given in [4]:

$$L_x = 3.34\sqrt{c}\sqrt{I_0/\rho}\,x. \tag{15}$$

Figure 2 shows streamlines calculated from (14), equal velocity lines calculated from (11), and boundaries of the active part of the jet, calculated from (12).

Solving (13) within the jet boundaries, we obtain the flow rate of liquid (or gas) passing through the cylindrical section of the active part of the jet (equal to the flow rate in the intersecting stream):

$$L_x' = L_x \operatorname{erf}(y_b/\sqrt{2}\,cx) - 2\pi xy_b U_*. \tag{16}$$

The flow rate reaches a maximum

$$(L_x')_{\max} = 1.44\,H\sqrt{I_0/\rho}$$

in the critical section with x value

$$x_{cr} = 3.75H.$$

Following the critical section the jet turns around. The turning center is located at the critical section (x_{cr} = 3.75 H; y_{cr} = 0.43 H).

Graphs of variation of flow rate of liquid (or gas) in a free jet and in a constrained jet, as calculated from (15) and (16), are presented in Fig. 3.

The transverse component of liquid (or gas) velocity at an arbitrary point of a constrained jet is given by

$$V = -\frac{1}{2\pi x}\frac{\partial\psi'}{\partial x} \tag{17}$$

or by

$$\frac{V}{U_x} = \frac{y}{x}\left[\exp\left(-\frac{1}{2}\frac{y^2}{c^2x^2}\right) - \exp\left(-\frac{1}{2}\frac{H^2}{c^2x^2}\right)\right] +$$

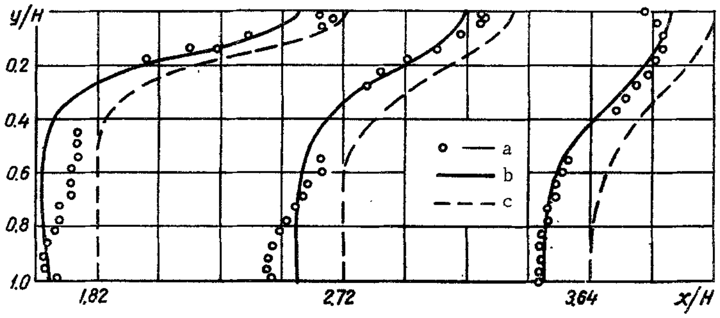

Fig. 4. Profiles of radial velocity components in cylindrical sections of a bounded volume: a—experimental; b—theoretical velocity profile U' in a constrained jet according to (7) with $U/(1/H)(I_0/\rho)^{1/2} = 1$; c—velocity U in a free jet, according to (2).

$$+\sqrt{\frac{\pi}{2}}\, c\left[\frac{y}{H}\,\mathrm{erf}\left(\frac{H}{\sqrt{2}\,cx}\right)-\mathrm{erf}\left(\frac{y}{\sqrt{2}\,cx}\right)\right]. \quad (18)$$

The limiting value of the relations obtained for constrained jets as $H \to \infty$ corresponds to the formulas given in [4] for free fan-shaped jets.

The material presented is confirmed, with some approximation, by tests of the author carried out in the Science Research Institute for Health Technology.

A jet of air of total flow rate $L_0 = 570$ m^3/hr was supplied from a nozzle of diameter 75 mm, the exit aperture of which was located in the center of the upper surface of the model of a room of dimensions 1500 × 1100 × 110 mm. A disk of diameter 112 mm was located at a distance of 30 mm from the nozzle. The airstream, issuing from the nozzle, encountered the disk, changed direction by 90°, flowing away to the side with an initial velocity $U_0 = 43$ m/sec, and flowed along the upper plate of the model, forming the active part of the jet. The intersecting stream was located at the lower plate of the model. The side boundaries of the model were made anechoic. The air passed out of the model through an aperture of diameter 200 mm at the center of the lower plate.

Figure 4 shows the profiles in reduced coordinates of the radial components of the air velocity.

NOTATION

U, U_x are the radial velocity components at an arbitrary point and on the axis of a free flat jet; U', V are the radial and transverse velocity components at an arbitrary point in a flat constrained jet; U_x', U_x'' are maximum velocities in the active part and in the intersecting stream of a constrained flat jet; U_* is the velocity shift in a constrained jet in comparison with a free jet; U_0 is the initial velocity of the jet; I_0 is the initial impulse of the jet, determined at uniform discharge from the formula $I_0 = \rho U_0 L_0$; ρ is the density of the liquid or gas; ψ is the stream function; L_x, L_x' are the flow rates of liquid or gas passing through a cylindrical section of the free and the constrained jet; L_0 is the initial flow rate in the jet; x is the radius of an arbitrary point in the stream; y is the ordinate of an arbitrary point in the stream; y_b is the ordinate of the boundary of the active part of the jet; H is the distance between planes; F is the area of a cylindrical section of the stream; c is the experimental constant.

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Institute of Health Technology, Moscow